# Understand Functions

💭 Think It Through

What is a function?

A teacher likes to remind his students that their future earnings are often a function of what they are learning.

What exactly does it mean to say that one thing is a function of another? It relates to dependence. Another way to present the idea above is, "What you are paid depends on what you know."

In mathematics, a **function** is a rule that defines a dependent relationship. A function creates exactly one **output**, or result, for each **input**. The diagram below shows the function "add 2."

Input = 3	→	Function: Add 2.	→	Output = 5
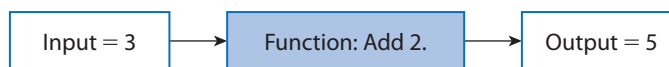
There is only one possible output for each input.

The function "add 2" is expressed in words. It can also be written as the equation $y = x + 2$, represented in a table of values, and shown as a graph.

Think What are some relationships that are functions?

Each coin of American currency is assigned one specific value in dollars. For example, the value of a penny is always $0.01. In this function, an ordered pair relates the name of a coin and its value in dollars.

Coin	Penny	Nickel	Dime	Quarter	Half-Dollar
Dollar Value	0.01	0.05	0.10	0.25	0.50

Most mathematical functions include ordered pairs of numbers. For example, a 120-pound person burns about 65 calories per mile while walking. The table below shows how many calories the person would burn walking different numbers of miles.

Miles (input)	1	2	3	4	5	6
Calories (output)	65	130	195	260	325	390

The input is the number of miles walked. The rule is to multiply the number of miles by 65. The output is the number of calories burned.

A basketball coach gives the starting players a game jersey. At the same time, he measures the players' heights. This relationship is a function. For each jersey number, he records only one player's height. If jersey number is the input and height is the output, then the relationship is a function.

Player's Jersey Number (input)	10	13	14	18	21
Player's Height in Inches (output)	68	73	75	68	74

> I see that the height 68 inches is paired with two different jersey numbers.

Now, reverse the relationship. What if player height is the input and jersey number is the output? The diagram below helps you see that when the input is 68, the output may be either 10 *or* 18. This is not a function. In a function, one input can have only one output.

Input: Height   Output: Jersey Number

68 → 10
73 → 13
74 → 14
75 → 18
     21
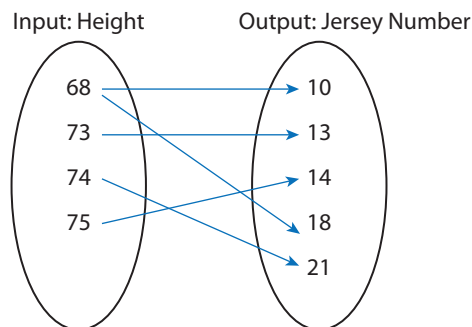
▶ Reflect

1. Do the data in this table show a function? If you switch the input and the output values, is it a function? Explain.

Input	3	3	5	5	6
Output	−3	2	4	5	6

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**Think About** ▶ **Identifying Functions**

🔍 **Let's Explore the Idea** Plot the ordered pairs from each table to represent the relationships as graphs. Then answer the problems.

2 The table below shows the number of dog licenses issued in the town of Palmer over a 5-year period. On the blank graph to the right, label and number the axes. Then plot the ordered pairs.

Year (input)	1	2	3	4	5
Number of Dog Licenses Issued (output)	75	100	125	125	150
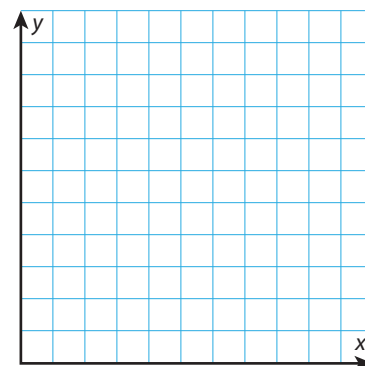
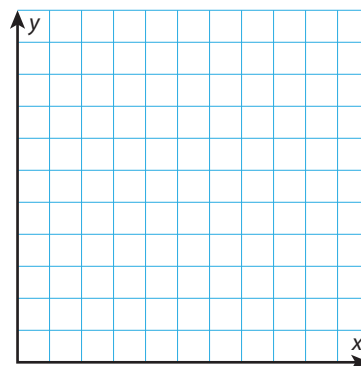**3** Describe the relationship between the input and the output values.

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4 Dogs age faster than humans do. Some people claim that dog years are a function of human years, as shown in the table. On the blank graph to the right, label and number the axes. Then plot the ordered pairs.

Age in Human Years (input)	1	2	3	4	5
Age in Dog Years (output)	7	14	21	28	35

5 Describe the relationship between the input and output values.

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Let's Talk About It
Answer the problems below as a group.

6 The relationships on the previous page are shown as graphs. Which graph or graphs show exactly one output for each input? Which set or sets of data represent a function?

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7 Think about the function related to dog licenses. Would there ever be one year that is assigned to two different quantities? Explain. _____

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8 Look at your answers to problems 3 and 5 on the previous page. Can you represent either of the functions with an equation? Explain. _____

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▶ **Try It Another Way** Substitute values into the equations to complete the tables. Then state whether the equation represents a function. Explain your reasoning.

9 $y = x + 2$

x (input)	−2	−1	0	1	2
y (output)					

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10 $y = 4x$

| x (input)	−2	−1	0	1	2
y (output)					

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Connect > Identifying Functions

Talk through these problems as a class, then write your answers below.

11 **Arrange** Use a diagram to represent a function. Put these numbers in the ovals to show ordered pairs that form a function: −4, −3, −3, −1, 1, 2, 5, 5, 6, 7. Complete the diagram that has been started. In the blank diagram, use the same numbers to show a different set of ordered pairs that form a function.

Input   Output   Input   Output

⁻4 ⟶ ⁻3
⁻3 ⟶ ⁻1
1
2
5
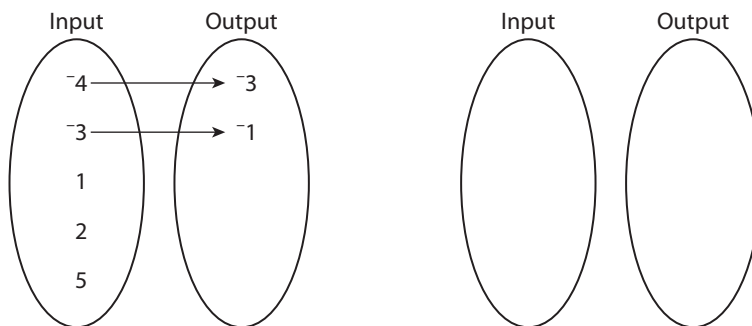
12 **Explain** A bag includes one number card each for the numbers 1–25. Each of 25 students randomly selects a number card from the bag. Carrie says that if the number is the input and the student is the output, the relationship is a function. Mario says that if the student is the input and the number is the output, the relationship is a function. Who is correct? Why? _____

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13 **Analyze** Each molecule of water contains 2 hydrogen atoms and 1 oxygen atom. Complete the table. Is the number of hydrogen atoms a function of the number of oxygen atoms? Explain.

Oxygen Atoms (Input)	1	2	3	4
Hydrogen Atoms (Output)	2			

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Apply ▶ **Identifying Functions**

14 **Put It Together** Sean and Rachel were both born on April 17. When Sean was 4, his sister Rachel was 2.

Part A Write an equation that can be used to determine Rachel's age given Sean's age. Write an equation that can be used to determine Sean's age given Rachel's age.

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Part B Complete the tables of values to show the relationship between their ages.

Sean's Age (input)	11	12	13	14	15	16
Rachel's Age (output)						

Rachel's Age (input)	1	2	3	4	5	6	7
Sean's Age (output)							

Part C Describe the relationships in the tables. Is either relationship a function? Explain.

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Part D The problem states that when Sean was 4, Rachel was 2. Sean's age is twice Rachel's age. Can this also be a rule for the relationship between their ages? Why or why not?

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